

# CASIO®

## SENIOR MATHEMATICS COMPETITION 2008

Final Round  
Friday August 15<sup>th</sup>

Time: 2 hours

### INSTRUCTIONS

1. *You may attempt all questions.*
2. *Calculators may be used.*
3. *Language dictionaries (non-electronic) are permitted, but no other reference material may be used.*
4. *Diagrams are not necessarily drawn to scale.*
5. *The mark allocated for each question is indicated. Marks will be awarded for clear reasoning; answers only will not necessarily earn full marks.*

*Total: 50 marks*

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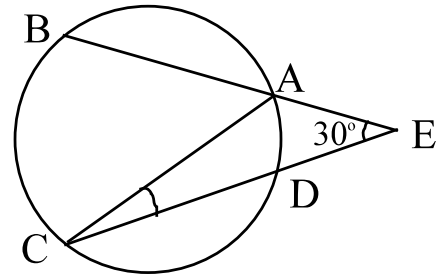


1. In the circle, arcs AB, BC and CD are all equal in length.

E is an external point and subtends an angle with B and C of  $30^\circ$ .

Find the size of angle ACE.

[3 marks]



2. A geometric sequence has the terms:  $a, ar, ar^2, ar^3, \dots$

Integers  $a, b$  and  $c$  are the first three terms of the geometric sequence with the property that  $a + b + c = 7$ .

How many such geometric sequences are there with this property?

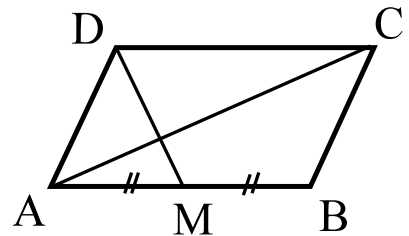
[6 marks]

3. ABCD is a parallelogram. M is the mid-point of AB.

The area of the parallelogram ABCD is  $A$  square units.

Find the areas of the four regions shown in the parallelogram.

[4 marks]



4. Two isosceles triangles are created with the following properties.

One has an apex angle of  $\alpha$ , two equal sides of length  $a$  and a base length  $b$ . ( $a \neq b$ )

The other has two base angles of  $\alpha$ , base length  $a$  and two equal sides of length  $b$ . ( $a \neq b$ )

Find the ratio  $\frac{a}{b}$ , and hence the size of angle  $\alpha$ .

[5 marks]

5.  $a, b$  and  $c$  are real numbers satisfying the expressions

$$a = x + \frac{1}{x}, \quad b = y + \frac{1}{y} \quad \text{and} \quad c = xy + \frac{1}{xy}$$

Prove that the value of the expression  $a^2 + b^2 + c^2 - abc$  is a constant.

[4 marks]

6.  $a$  and  $b$  are two real numbers.

Solve the pair of simultaneous equations.

$$a\sqrt{a} + b\sqrt{b} = 32$$

$$a\sqrt{b} + b\sqrt{a} = 31$$

[6 marks]

7. A square ABCD is inscribed by a circle, centre O as shown.

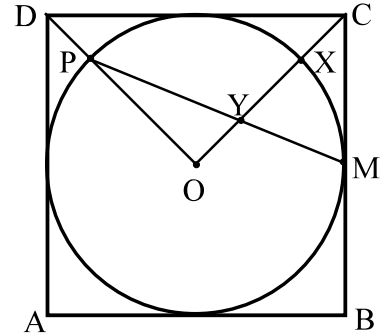
M is the mid-point of BC.

P is the point on the circle intersecting radius OD.

Y is the intersection of PM and the radius OC.

Prove length OY equals that of XC.

[5 marks]



8. At a movie theatre, the manager announces that a free ticket will be given to the first person in the line whose birthday is the same as someone in the line who has already bought a ticket. You have the option of joining the line in any position.

Assuming you don't know anyone else's birthday, and that birthdays are uniformly distributed throughout the 366 days of this year, what position in the line should you choose to give you the best chance of being the first duplicate birthday and getting the free ticket.

[5 marks]

9. Solve for  $x$ :  $(\ln x)^2 - 2 \cdot 5(\ln x)(\ln(4x - 5)) + (\ln(4x - 5))^2 = 0$

$x$  and all expressions are real.

Note:  $\ln x = \log_e x$

[4 marks]

10. A rectangular sheet of paper ABCD is folded through corner D so that corner A meets side BC at point E.

The areas of the three right angled triangles  $\triangle BEH$ ,  $\triangle ECD$  and  $\triangle DEH$  form an arithmetic sequence.

The area of the smallest triangle  $\triangle BEH$  is 1 square unit.

Find the dimensions of the rectangle ABCD.

[8 marks]

