



New Zealand  
Maths Olympiad Committee  
Intermediate problems  
Set 4

Remember that in all the following problems you are expected to provide a proof, that is, a complete and convincing argument of why your answer is correct. A simple answer, while a good start, is by no means enough!

1. Suppose that  $p$  and  $q$  are prime numbers and that the equation:

$$x^4 - px^3 + q = 0$$

has an integral root. What are the possible values of  $p$  and  $q$ ?

2. Feeling a bit bored, I added up the page numbers in a book of mine. I got the answer 2003. That didn't seem right, so I looked through the book again and noticed that one of the page numbers was repeated. Assuming that my arithmetic was right after all, how many pages did the book have and which page number was repeated?
3. In an acute angled triangle  $ABC$ , the altitude from  $A$  meets  $BC$  at  $A'$  and that from  $B$  meets  $AC$  at  $B'$ . If the length of  $A'B'$  is 24cm, and the length of  $AB$  is 26 cm, what is the length of the segment connecting the midpoint of  $A'B'$  to the midpoint of  $AB$ ?
4. A function  $f$  is defined on non-negative pairs of integers as follows:

$$\begin{aligned}f(0, n) &= n + 1 \\f(k, 0) &= f(k - 1, 1) \\f(k + 1, n + 1) &= f(k, f(k + 1, n)).\end{aligned}$$

Determine  $f(2, 1000)$ .