New Zealand
Maths Olympiad Committee
Intermediate problems
Set 2

Remember that in all the following problems you are expected to provide a proof, that is, a complete and convincing argument of why your answer is correct. A simple answer, while a good start, is by no means enough!

1. Determine all real solutions of the equation:

$$(x+1)^{2003} + (x+1)^{2002}(x-2) + (x+1)^{2001}(x-2)^2 + \cdots$$
$$\cdots + (x+1)^2(x-2)^{2001} + (x+1)(x-2)^{2002} + (x-2)^{2003} = 0.$$

2. A convex pentagon $ABCDE$ is given in which the areas of the triangles ABC, ABD, ACD, and ADE are all equal to the same value a. Determine the area of triangle BCE.

3. Prove that $n^n - n$ is a multiple of 24 for any odd positive integer n.

4. Eight friends each have a different amount of money (which is a whole number of dollars). Each person can distribute his/her money among the other seven (again in whole dollar amounts) so that they all wind up with the same amount of money. What is the smallest possible amount of money that the richest person in the group can have?