



New Zealand  
Maths Olympiad Committee  
Intermediate problems  
Set 1

Remember that in all the following problems you are expected to provide a proof, that is, a complete and convincing argument of why your answer is correct. A simple answer, while a good start, is by no means enough!

1. Find all positive integers which are equal to 11 times the sum of their digits.
2. What is the smallest value of  $w$  so that two non-overlapping circles of diameters 4 and 6 cm can be drawn in a 9 cm by  $w$  cm rectangle? (They are allowed to touch the edge, and each other, but not to overlap properly.)
3. Twelve people sat down around a circular table. Then they noticed that there were place-cards. Of course, no one was sitting in his/her correct spot. Show that no matter where they were supposed to sit, simply by having everyone shift left a certain number of places it can be arranged that at least *two* people are seated correctly.
4. Let  $c$  be some fixed positive integer, and consider the quadratic:

$$p(x) = cx^2 + x - c$$

For which values of  $c$  (if any) are the values of this quadratic at integer values  $x$  *never* multiples of 6?