

NZIMO 2003 Camp Problems I

1. Find all triples of positive integers (p, q, n) with p and q primes satisfying:

$$p(p+3) + q(q+3) = n(n+3).$$

2. Define a sequence of positive integer a_n by the formula:

$$a_n = n + \{\sqrt{n}\}$$

for positive integers $n \geq 1$, where $\{x\}$ is the rounded value of a real number x . Determine the smallest integer k such that the values

$$a_k, a_{k+1}, \dots, a_{k+2002}$$

form a sequence of 2003 consecutive integers.

3. Circle S lies inside circle T and touches it at A (this is the only point of contact between the two circles). From a point P , distinct from A , on T , chords PQ and PR of T are drawn which are tangent to S at X and Y respectively. Show that $\angle QAR = 2\angle XAY$.
4. The N positive integers 1 through N are arranged around a circle. The positive difference between each consecutive pair of values is computed and these values are summed. The total obtained is called the total variation of the arrangement. Find the minimum and maximum possible values of the total variation.