New Zealand Mathematical Olympiad Committee

2010 May Problems

These problems are intended to help students prepare for the 2010 camp selection problems, used to choose students to attend our week-long residential training camp in Christchurch in January.

In recent years the camp selection problems have been known as the "September Problems", as they were made available in September. This year we're going to trial moving the selection problems earlier in the year, releasing them in July and moving the due date to August. This will allow more time for pre-camp training, building up to Round One of the British Mathematical Olympiad in December.

The solutions will be posted in about two month's time, but can be obtained before then by email if you write to me with evidence that you've tried the problems seriously.

Good luck!

Chris Tuffley, 2010 NZ IMO team leader
c.tuffley@massey.ac.nz

1. Consider a parallelogram $ABCD$ such that the midpoint M of the side CD lies on the angle bisector of $\angle BAD$. Show that $\angle AMB$ is a right angle.

2. Let a be a positive integer, $a > 1$. Prove that, for every positive integer n, the number

$$n(2n+1)(3n+1)\cdots(an+1)$$

is divisible by all prime numbers less than a.

3. The first 2010 positive integers are arbitrarily ordered and written in a sequence. Then, to every number in the sequence, its position, expressed as a positive integer between 1 and 2010, is added. Prove that the new sequence contains two numbers whose difference is divisible by 2010.

4. Find all functions from the integers to the integers satisfying the following two conditions for all integers m, n:

   (a) $f(n)f(-n) = f(n^2)$

   (b) $f(m+n) = f(m) + f(n) + 2mn$.

May 25, 2010

www.mathsolympiad.org.nz