



SENIOR MATHEMATICS COMPETITION 2008

Preliminary round
Thursday 22 May 2008
Time allowed 1 ½ hours

Instructions

Attempt all questions. It is not expected that you will finish them all.

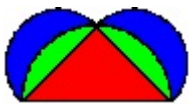
Full working should accompany all solutions.

Calculators may be used, but no other reference material is permitted.

Diagrams are not necessarily drawn to scale; they are presented as aids only.

Total: 50 marks

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and

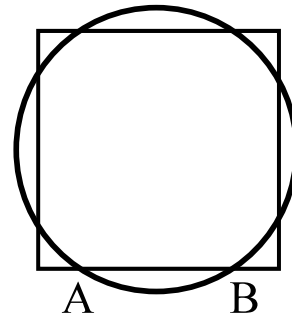


1. Find the value of

$$2008^2 - 2007^2 + 2006^2 - 2005^2 + \dots + 4^2 - 3^2 + 2^2 - 1^2 \quad [4]$$

2. A circle, of radius 1 cm, and a square are concentric.

If the area of circle and the square are equal, what is the length of the line segment AB?



[3]

3. Let f satisfy the functional equation

$$2f(x) + 3f\left(\frac{2x+29}{x-2}\right) = 100x + 140$$

Find $f(3)$

[5]

4. Three judges for a talent quest have to vote publicly on the three performers A, B and C, ranking their order of preference.

What is the probability that the judges vote so that two of them agree in their order of preference while the third differs?

[4]

5. Find the smallest positive integer n such that

$$\sqrt{n} - \sqrt{(n-1)} < \frac{1}{2008}$$

[4]

6. A triangle has sides of 19 cm, 20 cm and 13 cm. Find the length of the altitude perpendicular to the side of length 20 cm which divides the triangle into two equal area. [6]

7. A cubic equation has three roots which are perfect squares such that $a^2 + b^2 = c^2$, where a^2, b^2 and c^2 are the three roots.

If the equation is $x^3 + px^2 + qx + r = 0$, find the relationship that links p, q and r . [6]

8. Draw six cards from a standard pack of 52 playing cards without replacement. How many distinct ways can you choose these six cards so that all the following conditions are met.

- i) The first card drawn is a spade,
 - ii) the second card drawn is also a spade,
 - iii) the third card drawn is a club,
 - iv) the fourth card drawn is a diamond,
 - v) the fifth card drawn is a red card (either heart or diamond),
- and*
- vi) the last card drawn is an ace.

[7]

9. By definition, Euler's number (e) is defined as $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$ which is approximately equal to 2.7128182846.

What is the value of $\lim_{n \rightarrow \infty} \left(1 + \frac{2}{n}\right)^n$.

Derive your solution without using calculus, only algebra.

[6]

10. Find the limiting value of the series $\frac{3}{10} + \frac{21}{100} + \frac{117}{1000} + \frac{609}{10000} + \dots$

[5]